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Chapter 2

2.3 Vertex Form

Key Ideas

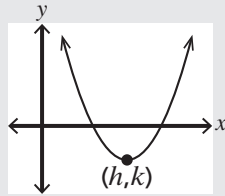
You have learned about the standard form and factored form of quadratic relations. In the standard form, the y -intercept and direction of opening can be easily identified, whereas in the factored form, the zeros can be found by inspection.

The vertex form is another common form of quadratic relations.

Vertex Form

$$y = a(x - h)^2 + k$$

vertex: (h, k)



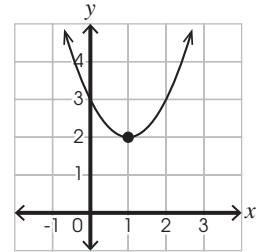
The vertex form allows you to find the vertex easily. It is also commonly used to represent a quadratic relation when performing transformations, which you will learn in Chapter 3.

Examples

Identify the vertex of each quadratic relation.

$$y = (x - 1)^2 + 2$$

vertex: $(1, 2)$

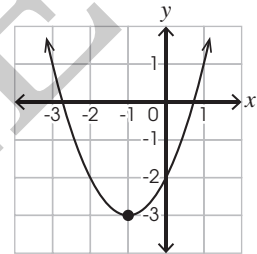


Be mindful of the values of h and k . Make sure that they have the correct signs.

$$y = (x + 1)^2 - 3$$

$$= (x - (-1))^2 + (-3)$$

vertex: $(-1, -3)$



Identify the vertex and find the y -intercept of each quadratic relation. Then write the letters in the circles to match.

Try these!

①

A $y = (x - 1)^2 + 1$

a. vertex:

$(1, \quad)$

b. y -intercept:

$$y = (0 - \quad)^2 + 1$$

$$y = \quad$$

B $y = -2(x - 1)^2 + 1$

a. vertex:

$(\quad, 1)$

b. y -intercept:

$$y = -2(0 - 1)^2 + \quad$$

$$y = \quad$$

C $y = (x + 1)^2 - 1$

a. vertex:

(\quad, \quad)

b. y -intercept:

$$y = \quad$$

$$y = \quad$$

D $y = -\frac{1}{2}(x + 1)^2 - 1$

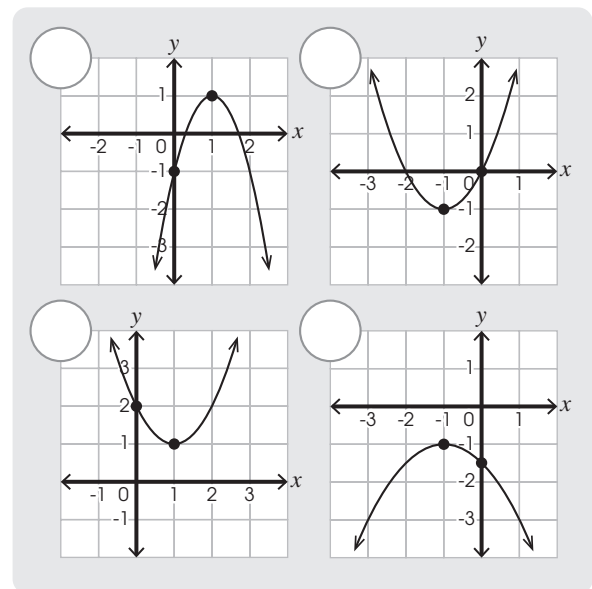
a. vertex:

(\quad, \quad)

b. y -intercept:

$$y = \quad$$

$$y = \quad$$



Find the vertex, y-intercept, and direction of opening of each quadratic relation. Then label its graph.

② $y = (x - 3)^2 - 5$

- a. vertex: _____
 b. y-intercept: _____
 c. direction of opening:

③ $y = -\frac{1}{3}(x + 3)^2$

- a. vertex: _____
 b. y-intercept: _____
 c. direction of opening:

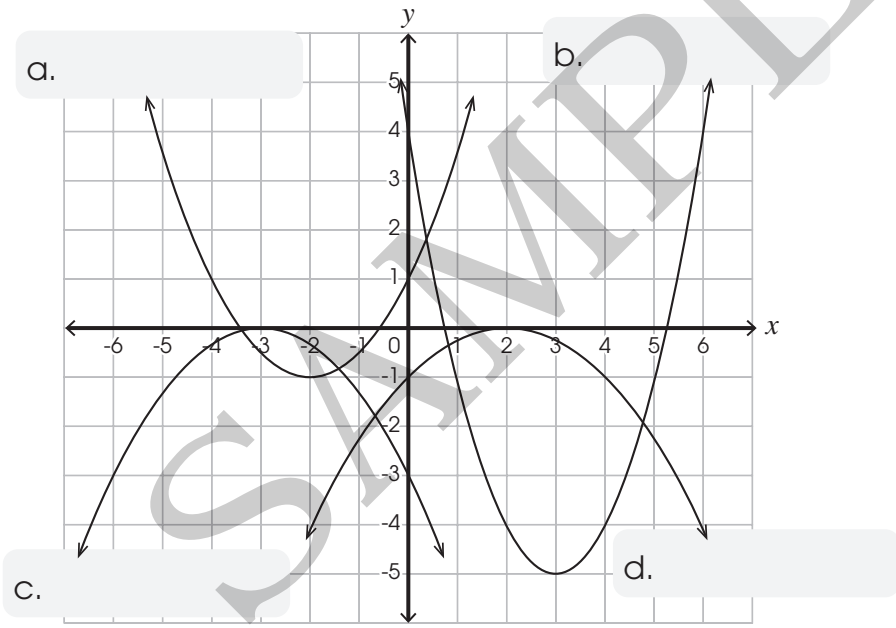
④ $y = \frac{1}{2}(x + 2)^2 - 1$

- a. vertex: _____
 b. y-intercept: _____
 c. direction of opening:

⑤ $y = -\frac{1}{4}(x - 2)^2$

- a. vertex: _____
 b. y-intercept: _____
 c. direction of opening:

⑥



Hint



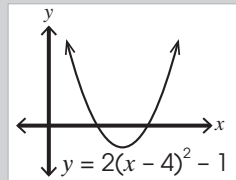
To determine whether a quadratic relation opens upward or downward, identify the value of a in its vertex form.

$y = a(x - h)^2 + k$

if $a > 0$, opens upward
 if $a < 0$, opens downward

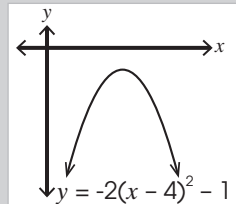
e.g. $y = 2(x - 4)^2 - 1$

> 0 ; upward



$y = -2(x - 4)^2 - 1$

< 0 ; downward



Find the features of the parabola of each quadratic relation written in vertex form.

⑦

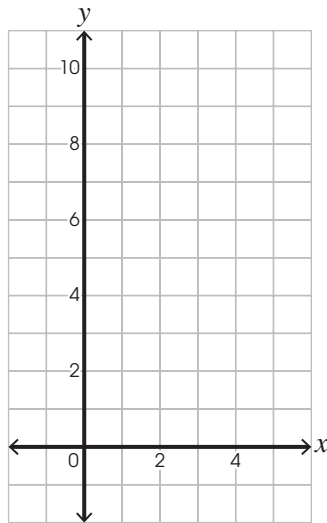
quadratic relation	vertex	y-intercept	direction of opening	axis of symmetry	optimal value	max. or min.
$y = (x - 2)^2 + 1$						
$y = -(x + 3)^2 - 2$						
$y = -3(x - 5)^2 + 10$						
$y = \frac{1}{4}(x + 6)^2$						

Complete the table of values and graph each relation. Find the vertex. Then answer the question.

⑧ $y = (x - 2)^2 + 1$

x	y
-1	
0	
1	
2	
3	
4	
5	

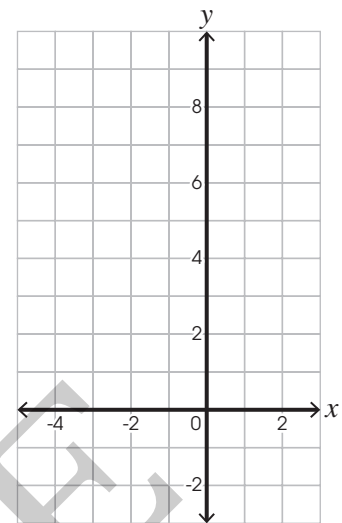
vertex



⑨ $y = (x + 1)^2 - 2$

x	y
-4	
-3	
-2	
-1	
0	
1	
2	

vertex



- ⑩ Relate the parameters, h and k , and the vertex of each relation. What do you find?

$$y = (x - h)^2 + k$$

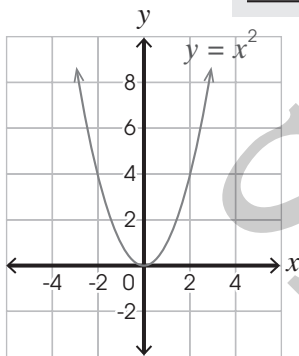
↑
translates
left or right

↑
translates
up or down

Identify and write the vertex of each relation. Then sketch its graph.

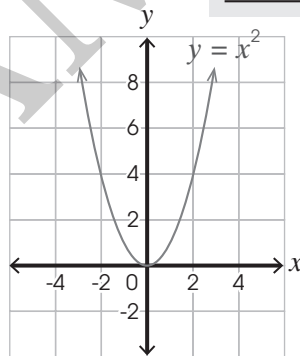
⑪ $y = (x + 2)^2 - 1$

vertex



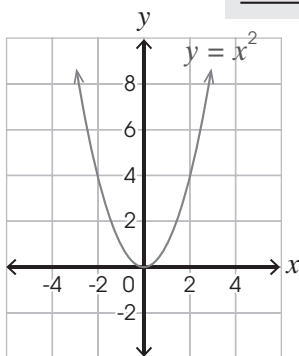
⑫ $y = (x - 2)^2 - 1$

vertex



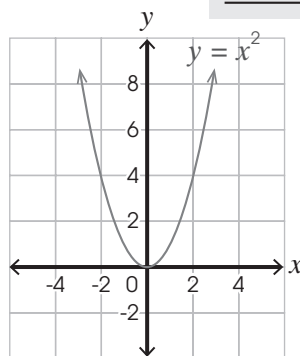
⑬ $y = (x - 1)^2 + 2$

vertex



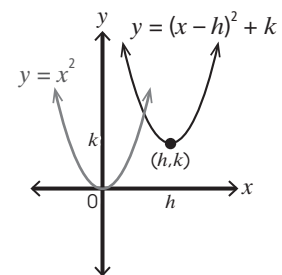
⑭ $y = (x - 1)^2 - 2$

vertex



Hint

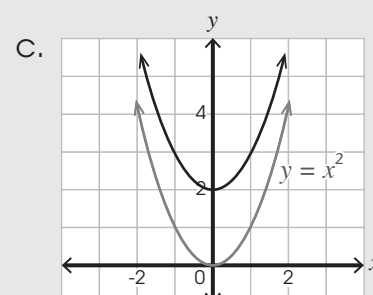
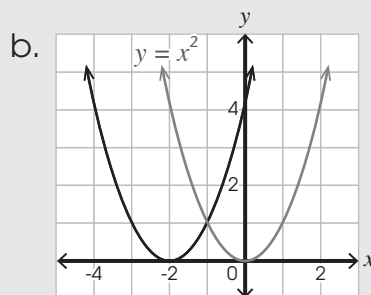
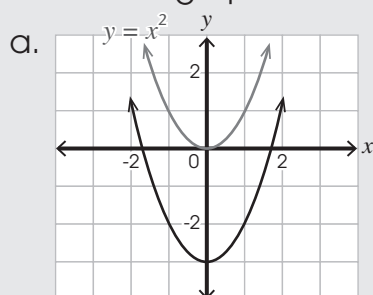
$$y = (x - h)^2 + k$$



(h, k) is the vertex of $y = (x - h)^2 + k$.

Answer the questions.

- ⑮ The graph of $y = x^2$ is translated as shown. Find the quadratic relations of the translated graphs.



- ⑯ Match each quadratic relation with its graph.

a. $y = (x + 3)^2 - 1$

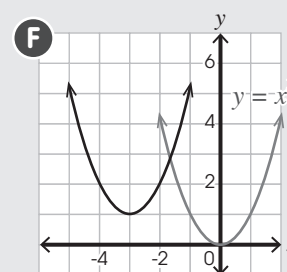
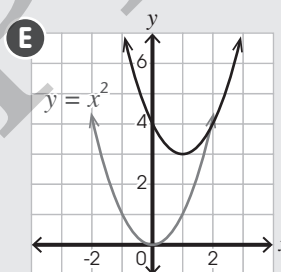
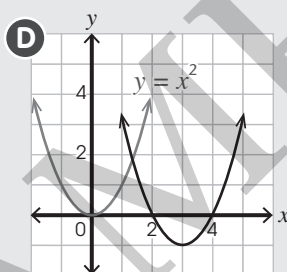
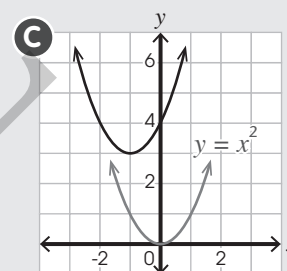
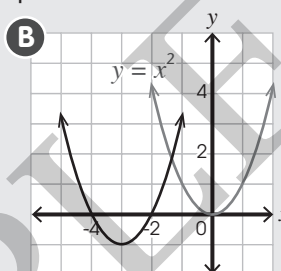
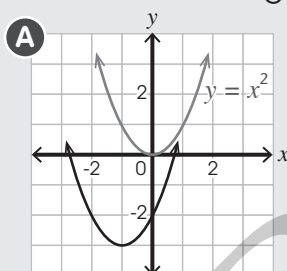
b. $y = (x + 3)^2 + 1$

c. $y = (x + 1)^2 + 3$

d. $y = (x - 1)^2 + 3$

e. $y = (x + 1)^2 - 3$

f. $y = (x - 3)^2 - 1$



- ⑰ Describe the transformations of $y = x^2$ represented by each relation. Then sketch its graph. Make sure to start with the graph of $y = x^2$.

a. $y = x^2 - 5$

b. $y = (x + 4)^2$

c. $y = (x - 2)^2 + 4$

d. $y = (x + 3)^2 - 2$

- ⑱ The graph of $y = x^2$ is translated as described below. Write the quadratic relation of each translated graph.

a. translated 5 units right and 1 unit down

b. translated 4 units left and 3 units up



The order of the transformations does not matter.

MATH IRL

Quadratic equations are used by engineers of various fields. When developing equipment that has the resemblance of a curve, such as auto bodywork, quadratic equations are used to model them. Automobile engineers utilize quadratic equations to build the optimal brake system as well. Scan this QR code to learn more about the application of quadratic equations in different fields.

